Online Appendix for

Supply Chain Resilience: Should Governments Promote International Diversification or Reshoring?

By

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We provide in this appendix derivations of expressions discussed in the main text, as well as proofs of arguments that are not shown there. The appendix is organized according to sections in the body of the paper in order to make it easy for the reader to find these items.

Section 2

Section 2.4

We begin by deriving expected profits $\Pi_j := \Pi_j(\boldsymbol{\mu})$, $j \in \{f, h, b\}$, where $\boldsymbol{\mu} := (\mu_h, \mu_f, \mu_b)$. To this end, recall the profit function (7),

$$\pi^{J,K}(\omega) = \frac{s\left[p(\omega)/A\right]}{\sigma\left[p(\omega)/A\right]}(P^J)^{1-\varepsilon},$$

for $J \in \{H, J, B\}$ the state of the world and $K \in \{H, F\}$ the country from which the input is supplied from. Given a state J, all firms sourcing from the same location choose the same prices. In states H and F, only sourcing from one country is feasible, and we use $\pi^J := \pi^{J,J}$ to denote the realized profits in state J for firms that have an active supply chain in country J.

First consider a state $J \in \{H, F\}$ in which supply chains from one country are disrupted but not so in the other country. In such a state, only firms that adopted a strategy of investing only in country J and those that invested in both countries might be able to produce, provided that their bilateral relations do not suffer an idiosyncratic shock. Each such firm pays q_J for its input. In this case, the market clearing condition (4) becomes

$$1 \equiv n^J(\boldsymbol{\mu})\, s[z^J(\boldsymbol{\mu})], \qquad J \in \{H, F\}\ , \tag{20}$$

where

$$n^J(\boldsymbol{\mu}) = (\mu_j + \mu_b)\,\rho$$

and $z^J = p^J/A^J$. These equations yield relative prices z^J in state $J \in \{H, F\}$ as functions of $\boldsymbol{\mu}$, denoted $z^J(\boldsymbol{\mu})$.

Next note that, in state $J \in \{H, F\}$, the price index (5) can be expressed as

$$\log P^J = C_P + \log \frac{p^J}{z^J} - n^J \int_{z^J}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta,$$

where from (6),

$$p^J = \frac{\sigma\left(z^J\right)}{\sigma\left(z^J\right) - 1} q_J.$$

Using the function $z^J(\boldsymbol{\mu})$ and (20), we can express the price index P^J as a function of $z^J(\boldsymbol{\mu})$,

$$\log P^J\left[z^J(\boldsymbol{\mu})\right] := C_P + \log \frac{\sigma\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right] - 1} + \log \frac{q_J}{z^J(\boldsymbol{\mu})} - \frac{1}{s\left[z^J(\boldsymbol{\mu})\right]} \int_{z^J(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta,\ J \in \{H, F\}. \quad (21)$$

This function, together with (7) and $z^J(\boldsymbol{\mu})$, can be used to compute the profits of an active firm in state J, which are

$$\pi^J\left[z^J(\boldsymbol{\mu})\right] := \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right]} P^J\left[z^J(\boldsymbol{\mu})\right]^{1-\varepsilon}, \quad J \in \{H, F\}. \quad (22)$$

The functions $P^J(z)$ and $\pi^J(z)$, defined in (21) and (22), are decreasing in z. To see this, differentiate $\log P^J(z)$ with respect to z, which yields[29]

$$\frac{1}{P^J(z)} \frac{dP^J(z)}{dz} = -\frac{\sigma'(z)}{\sigma(z)\left[\sigma(z) - 1\right]} + \frac{s'(z)}{s(z)^2} \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta < 0,\ J \in \{H, F\}. \quad (23)$$

Next, differentiate $\log \pi^J(z)$ with respect to z, which gives

$$\frac{1}{\pi^J(z)} \frac{d\pi^J(z)}{dz} = -\frac{\sigma'(z)}{\sigma(z)} \frac{\sigma(z) - \varepsilon}{\sigma(z) - 1} + \frac{s'(z)}{s(z)} \left[1 - \frac{\varepsilon - 1}{s(z)} \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right],\ J \in \{H, F\}. \quad (24)$$

Equation (3) in the main text implies (see Matsuyama and Ushchev (2020)):

$$\frac{s(\zeta)}{\zeta} = \frac{s'(\zeta)}{1 - \sigma(\zeta)}.$$

Therefore

$$\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta = \int_{z^J}^{\bar{z}} \frac{-s'(\zeta)}{\sigma(\zeta) - 1} d\zeta < \int_{z^J}^{\bar{z}} \frac{-s'(\zeta)}{\sigma(z^J) - 1} d\zeta = \frac{s(z) - s(\bar{z})}{\sigma(z) - 1} = \frac{s(z)}{\sigma(z) - 1}. \quad (25)$$

Using this inequality, we obtain

$$\frac{1}{\pi^J(z)} \frac{d\pi^J(z)}{dz} < \left[\frac{s'(z)}{s(z)} - \frac{\sigma'(z)}{\sigma(z)}\right] \frac{\sigma(z) - \varepsilon}{\sigma(z) - 1} < 0,\ J \in \{H, F\},$$

which we summarize in the following

[29] Recall that $\sigma(z) > \varepsilon$ at our equilibrium points while $\sigma'(z) \geq 0$ and $s'(z) < 0$.

Lemma 1 *The functions $P^J(z)$ and $\pi^J(z)$ are declining in z for $J \in \{H, F\}$.*

In state B, in which supply chains from both countries are viable, diversified firms prefer to source from the cheaper country F (recall that $q_F < q_H$), if they can. In this case, the number of firms that source from F and pay q_F for their inputs is $n^{B,F}(\boldsymbol{\mu}) = (\mu_f + \mu_b)\rho$. The number of firms that source from country H and pay q_H for inputs is $n^{B,H}(\boldsymbol{\mu}) = \mu_h \rho + \mu_b \rho (1-\rho)$. The market clearing condition (4) implies

$$1 \equiv n^{B,H}(\boldsymbol{\mu})\, s[z^{B,H}(\boldsymbol{\mu})] + n^{B,F}(\boldsymbol{\mu}) s[z^{B,F}(\boldsymbol{\mu})]\ , \tag{26}$$

which is equation (14) in the main text. The pricing equation (6) implies

$$\frac{z^{B,H}(\boldsymbol{\mu})}{z^{B,F}(\boldsymbol{\mu})} \equiv \left\{ \frac{\sigma[z^{B,H}(\boldsymbol{\mu})]}{\sigma[z^{B,H}(\boldsymbol{\mu})]-1} \right\} \Big/ \left\{ \frac{\sigma[z^{B,F}(\boldsymbol{\mu})]}{\sigma[z^{B,F}(\boldsymbol{\mu})]-1} \right\} \frac{q_H}{q_F}\ . \tag{27}$$

From here, we obtain solutions to the relative prices $z^{B,i}$, $i = H, F$, as functions of the vector $\boldsymbol{\mu}$, $z^{B,i}(\boldsymbol{\mu})$, $i \in \{H, F\}$. Furthermore, equation (27) implies that prices of the goods produced with inputs from country F are strictly cheaper than goods produced with inputs from country H in state B. To see this, suppose not, such that $p^{B,H} \leq p^{B,F}$ and therefore $z^{B,H} \leq z^{B,F}$. Equation (27) then returns

$$p^{B,H} = \left(\frac{\sigma(z^{B,H})}{\sigma(z^{B,H})-1} \right) q_H \leq \left(\frac{\sigma(z^{B,F})}{\sigma(z^{B,F})-1} \right) q_F = p^{B,F} < \left(\frac{\sigma(z^{B,F})}{\sigma(z^{B,F})-1} \right) q_H .$$

However, the mark-up function $z \to \sigma(z)/(\sigma(z)-1)$ is (weakly) increasing in p under Assumption 2, thus contradicting the strict inequality above. It follows that, for any vector $\boldsymbol{\mu}$, we have $z^{B,H}(\boldsymbol{\mu}) > z^{B,F}(\boldsymbol{\mu})$.

To derive the price index (5) for state B, first note that the pricing equation (6) implies

$$\frac{1}{A^B(\boldsymbol{\mu})} = \frac{z^{B,i}(\boldsymbol{\mu}) \left\{ \sigma[z^{B,i}(\boldsymbol{\mu})] - 1 \right\}}{q_i \sigma[z^{B,i}(\boldsymbol{\mu})]}, \quad i \in \{H, F\}.$$

Using (14), we can write

$$\log A^B(\boldsymbol{\mu}) = \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}) s\left[z^{B,i}(\boldsymbol{\mu}) \right] \log \left\{ \frac{q_i}{z^{B,i}(\boldsymbol{\mu})} \frac{\sigma\left[z^{B,i}(\boldsymbol{\mu}) \right]}{\sigma\left[z^{B,i}(\boldsymbol{\mu}) \right] - 1} \right\}.$$

Now, the price index (5) can be expressed as

$$\log P^B(\boldsymbol{\mu}) := \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu})\, s\left[z^{B,i}(\boldsymbol{\mu}) \right] \log P^B\left[z^{B,i}(\boldsymbol{\mu}) \right], \tag{28}$$

where the function $\log P^J(z)$ is defined in (21). Using this result for the price index, profits of a firm that sources from country J in state B amount to

$$\pi^{B,i}(\boldsymbol{\mu}) := \frac{s\left[z^{B,i}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,i}(\boldsymbol{\mu})\right]} P^{B}(\boldsymbol{\mu})^{1-\varepsilon}, \quad i \in \{H, F\}. \tag{29}$$

Now consider expected profits from strategy j, Π_j, $j \in \{h, f, b\}$. For a firm that invests in a single supply chain, expected profits are

$$\Pi_h = \delta^H \pi^H \rho + \delta^B \pi^{B,H} \rho - k,$$

$$\Pi_f = \delta^F \pi^F \rho + \delta^B \pi^{B,F} \rho - k,$$

where δ^J is the probability that only supply chains from country J will be available, $J \in \{H, F\}$, and δ^B is the probability that supply chains from both countries will be available. These probabilities are $\delta^H = \gamma_H (1 - \gamma_F)$, $\delta^F = \gamma_F (1 - \gamma_H)$, and $\delta^B = \gamma_F \gamma_H$. Using the profit functions (22) and (29), this yields

$$\Pi_h = \Pi_h(\boldsymbol{\mu}) := \delta^H \frac{s\left[z^H(\boldsymbol{\mu})\right]}{\sigma\left[z^H(\boldsymbol{\mu})\right]} P^H\left[z^H(\boldsymbol{\mu})\right]^{1-\varepsilon} \rho + \delta^B \frac{s\left[z^{B,H}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,H}(\boldsymbol{\mu})\right]} P^B(\boldsymbol{\mu})^{1-\varepsilon} \rho - k, \tag{30}$$

$$\Pi_f = \Pi_f(\boldsymbol{\mu}) := \delta^F \frac{s\left[z^F(\boldsymbol{\mu})\right]}{\sigma\left[z^F(\boldsymbol{\mu})\right]} P^F\left[z^F(\boldsymbol{\mu})\right]^{1-\varepsilon} \rho + \delta^B \frac{s\left[z^{B,F}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,F}(\boldsymbol{\mu})\right]} P^B(\boldsymbol{\mu}, \boldsymbol{q})^{1-\varepsilon} \rho - k. \tag{31}$$

For a firm that invests in supply chains in both countries, expected profits are

$$\Pi_b = \sum_{J=H,F} \delta^J \pi^J \rho + \delta^B\left[\pi^{B,F}\rho + \pi^{B,H}(1-\rho)\rho\right] - 2k.$$

A firm that adopts this strategy expects profits π^F if the supply chains survive only in country F, provided it does not suffer an idiosyncratic disruption there. Similarly, it expects profits π^H if the supply chains survive only in country H, provided it does not suffer an idiosyncratic disruption there. In case supply chains in both countries are viable, the firm expects profits $\pi^{B,F}$ if its bilateral relation survives in country F and profits $\pi^{B,H}$ if its bilateral relation in F does not survive but that in H does survive. Using (22) and (29), this yields

$$\begin{aligned}\Pi_b = \Pi_b(\boldsymbol{\mu}) := &\sum_{J=H,F} \delta^J \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right]} P^J\left[z^J(\boldsymbol{\mu})\right]^{1-\varepsilon} \rho \\ &+ \delta^B \left\{\frac{s\left[z^{B,F}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,F}(\boldsymbol{\mu})\right]} + \frac{s\left[z^{B,H}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,H}(\boldsymbol{\mu})\right]}(1-\rho)\right\} P^B(\boldsymbol{\mu})^{1-\varepsilon} \rho - 2k. \end{aligned} \tag{32}$$

Using these functions and $P^J(\boldsymbol{\mu}) := P^J\left[z^J(\boldsymbol{\mu})\right]$, $J \in \{H, F\}$, we obtain the welfare function (11) in the main text.

Section 2.6

Armed with these expressions, we now prove Figure 1. Suppose that $q_F \nearrow q_H$ and $\gamma_F < \gamma_H$. In this partially asymmetric world, given an aggregate state J, the country from which the input is supplied is irrelevant. In particular, in state B, equation (27) dictates that $z^{B,F} \to z^{B,H}$. As in state H and F, we can then use the notation $z^B(\boldsymbol{\mu}) := z^{B,H}(\boldsymbol{\mu}) = z^{B,F}(\boldsymbol{\mu})$. Consequently, only the total number of products available in state B matters, $n^B(\boldsymbol{\mu}) := n^{B,F}(\boldsymbol{\mu}) + n^{B,H}(\boldsymbol{\mu}) = \rho(1 + (1-\rho)(1-\mu_h-\mu_f))$, and the market clearing condition (26) rewrites $1 = n^B(\boldsymbol{\mu})s[z^B(\boldsymbol{\mu})]$.

Inasmuch as country H is safer than country F, relatively more firms want to settle a single supply chain in H than in F. To see this, suppose the contrary: firms invest relatively more in the risky country, $\mu_f > 0$ and $\mu_f \geq \mu_h$. From the market clearing conditions, we then have

$$s[z^F(\boldsymbol{\mu})] = \frac{1}{\rho(\mu_f + \mu_b)} \leq \frac{1}{\rho(\mu_h + \mu_b)} = s[z^H(\boldsymbol{\mu})].$$

Since $s'(z) < 0$, it must be that $z^H(\boldsymbol{\mu}) \leq z^F(\boldsymbol{\mu})$. But $\pi'(z) < 0$, so that $\delta^F\pi[z^F(\boldsymbol{\mu})) \leq \delta^F\pi[z^H(\boldsymbol{\mu})] < \delta^H\pi[z^H(\boldsymbol{\mu})]$. However, this in turn implies that the expected profits of the foreign strategy are lower, $\Pi_f < \Pi_h$, and therefore $\mu_f = 0$, a contradiction. Hence, in equilibrium, it must either be that no firms invest in the risky country, $\mu_f = 0$, or if some firms do, relatively more firms need to invest in the safe country, $\mu_f > 0$ and $\mu_h > \mu_f$. Accordingly, it must also be that the expected profits of the safer strategy are (weakly) higher than the expected profits of the less safe strategy, $\Pi_h \geq \Pi_f$.

Given that the expected profits of the home supply chain are weakly higher than those with a supplier in the foreign country, firms' investments are dictated by two comparisons: home versus foreign supply chains, $\Pi_h(\mu_h, \mu_f, \mu_b) - \Pi_f(\mu_h, \mu_f, \mu_b)$, and single supply chain at home versus diversification, $\Pi_h(\mu_h, \mu_f, \mu_b) - \Pi_b(\mu_h, \mu_f, \mu_b)$.[30] Using the expressions for expected profits (30:32), these two optimality conditions respectively read

$$\Pi_h(\mu_h, \mu_f, \mu_b) \geq \Pi_f(\mu_h, \mu_f, \mu_b) \quad \iff \quad \delta^H\pi[z^H(\mu_h, \mu_f, \mu_b)] \geq \delta^F\pi[z^F(\mu_h, \mu_f, \mu_b)],$$

and

$$\Pi_b(\mu_h, \mu_f, \mu_b) \geq \Pi_h(\mu_h, \mu_f, \mu_b) \quad \iff \quad \delta^F\pi[z^F(\mu_h, \mu_f, \mu_b)] + \delta^B\rho(1-\rho)\pi[z^B(\mu_h, \mu_f, \mu_b)] \geq k.$$

In addition, profits must be positive, $\Pi_j(\mu_h, \mu_f, \mu_b) \geq 0$ for $j \in \{h, f, b\}$. These three conditions together dictate the features of the equilibrium.

Figure 1 depicts the fraction of firms choosing each strategy as a function of k when profits are unbounded; that is when $\lim_{z\to 0^+} \pi(z) = \infty$. We make this assumption throughout the proof, and come back to the case of bounded profit at the end of the section.

[30] Without $\Pi_h \geq \Pi_f$, we would also need to compare $\Pi_f - \Pi_b$.

Existence of k_1 For $k \to 0^+$, investing in resilience is clearly the most profitable option, $\Pi_b > \Pi_h > \Pi_f$, where the second inequality follows from $\delta^H \pi[z^H(0,0,1)] > \delta^F \pi[z^F(0,0,1)]$. Hence, for low k, $\mu_h = 0 = \mu_f$ and $\mu_b = 1$. As the fixed cost increases, the gap between $\Pi_h(0,0,1) - \Pi_b(0,0,1)$ shrinks to the point where the two strategies yield the same expected profits. This occurs at k_1, defined by

$$k_1 := \delta^F \rho \pi[z^F(0,0,1)] + \delta^B \rho(1-\rho)\pi[z^B(0,0,1)].$$

Furthermore, at k_1, excepted profits of strategy h reads

$$\Pi_h = \delta^B \rho^2 \pi^B[z^B(0,0,1)] + \rho\left(\delta^H \pi[z^H(0,0,1)] - \delta^F \pi[z^F(0,0,1)]\right) > 0,$$

where the inequality follows from $z^H(0,0,1) = z^F(0,0,1)$ and $\delta^H > \delta^F$. Hence, $\Pi_b > 0$ for all $k \in [0, k_1]$.

Existence of k_2 At $k = k_1$, we thus have $\Pi_b(0,0,1) = \Pi_h(0,0,1) > \Pi_f(0,0,1)$. For $k \in B^+(k_1)$, $\Pi_b(0,0,1) < \Pi_h(0,0,1)$, such that $(0,0,1)$ cannot be an equilibrium.[31] Instead, it must be that $\mu_h > 0$ and $\mu_b = 1 - \mu_h > 0$. When that is the case, firms must be indifferent between the two strategies and must prefer them to the offshoring strategy,

$$\begin{aligned}
k &= \delta^F \rho \pi[z^F(\mu_h, 0, 1-\mu_h)] + \delta^B \rho(1-\rho)\pi[z^B(\mu_h, 0, 1-\mu_h)], \\
0 &> \delta^F \pi[z^F(\mu_h, 0, 1-\mu_h)] - \delta^H \pi[z^H(\mu_h, 0, 1-\mu_b)], \\
\mu_f &= 0,\ \mu_b = 1-\mu_h > 0.
\end{aligned}$$

First, note that the first and second conditions imply that expected profits are positive. For instance, the expected profits of strategy h is given by

$$\Pi_h = \delta^B \rho^2 \pi[z^B(\mu_h, 0, 1-\mu_h)] + \rho\left\{\delta^H \pi[z^H(\mu_h, 0, 1-\mu_h)] - \delta^F \rho \pi[z^F(\mu_h, 0, 1-\mu_h)]\right\} > 0,$$

where the first equality follows from the first condition and the inequality follows from the second condition. Second, when $\mu_h > 0$ and $\mu_b = 1 - \mu_h > 0$, the market clearing conditions in states F, H and B read $s[z^F(\boldsymbol{\mu})]\rho(1-\mu_h) = 1$, $s[z^H(\boldsymbol{\mu})]\rho = 1$ and $s[z^B(\boldsymbol{\mu})]\rho[1 + (1-\mu_h)(1-\rho)] = 1$ respectively. Hence, $\pi[z^F(\boldsymbol{\mu})]$ and $\pi[z^B(\boldsymbol{\mu})]$ are increasing in μ_h, while $\pi[z^H(\boldsymbol{\mu})]$ is independent of $\boldsymbol{\mu}$. This implies that μ_h is increasing in k for $k \in B^+(k_1)$. Finally, unbounded profits imply that there exists a $\bar{\mu}_h^2$ such that the second condition cannot hold for any $\mu_h > \bar{\mu}_h^2$.[32] This limiting μ_h is defined by

$$\delta^H \pi(\bar{z}) = \delta^F \pi[z^F(\bar{\mu}_h^2, 0, 1-\bar{\mu}_h^2)].$$

[31] The correspondence $B^+ : \mathbb{R} \mapsto 2^{\mathbb{R}}$ is defined as $B^+(x) = [x, x+\vartheta)$ for $\vartheta > 0$ small.

[32] Otherwise, the equality would hold for $\mu_h \nearrow 1 \iff z^F \searrow 0$, at which point $\pi(z^F) \to \infty > \pi[z^H(1,0,0)]$, a contradiction.

Accordingly, we define the second fixed cost threshold k_2 as

$$k_2 = \delta^F \rho\pi[z^F(\bar{\mu}_h^2, 0, 1-\bar{\mu}_h^2)] + \delta^B \rho(1-\rho)\pi[z^B(\bar{\mu}_h^2, 0, 1-\bar{\mu}_h^2)].$$

For all $k \in B^+(k_2)$, we would have $\mu_h > \bar{\mu}_h^2$, which in turn would imply $\Pi_f > \Pi_h$ so that $(\mu_h, 0, 1-\mu_h)$ cannot be an equilibrium allocation.

Existence of k_3 For $k \in B^+(k_2)$, the equilibrium must be of the type $\mu_h > 0$, $\mu_f > 0$ and $\mu_b = 1-\mu_h-\mu_f > 0$. When that is the case, it must be that $\Pi_b = \Pi_h = \Pi_f$, or

$$\begin{aligned}
k &= \delta^F \rho\pi[z^F(\boldsymbol{\mu})] + \delta^B \rho(1-\rho)\pi[z^B(\boldsymbol{\mu})],\\
0 &= \delta^H[z^H(\boldsymbol{\mu})] - \delta^F \pi[z^F(\boldsymbol{\mu})],\\
1 &\geq \mu_h + \mu_f(\mu_h).
\end{aligned}$$

As before, note that the first and second conditions together imply that the expected profits of the three strategies are positive. The market clearing condition in each state are $s[z^F(\boldsymbol{\mu})]\rho(1-\mu_h) = 1$, $s[z^H(\boldsymbol{\mu})]\rho(1-\mu_f) = 1$, and $s[z^B(\boldsymbol{\mu})]\rho[1+(1-\rho)(1-\mu_f-\mu_h)] = 1$. In particular, z^F, z^H and z^B are only functions of μ_h, μ_f and $\mu_h+\mu_f$ respectively. Accordingly, let $\tilde{z}(\mu)$ and $\tilde{z}^B(\mu)$ be defined respectively by $s[\tilde{z}(\mu)]\rho(1-\mu) = 1$ and $s[\tilde{z}^B(\mu)]\rho(1+(1-\rho)(1-\mu)) = 1$. The first two equilibrium conditions then rewrite

$$\begin{aligned}
k &= \delta^F \rho\pi[\tilde{z}(\mu_h)] + \delta^B \rho(1-\rho)\pi[\tilde{z}^B(\mu_h+\mu_f)],\\
0 &= \delta^H \pi[\tilde{z}(\mu_f)] - \delta^F \pi[\tilde{z}(\mu_h)].
\end{aligned}$$

Both $\tilde{z}$ and $\tilde{z}^B$ are decreasing functions, such that $d\pi[\tilde{z}(\mu)]/d\mu > 0$. The second condition thus implies that μ_f is increasing in μ_h, and the first condition implies that μ_h is increasing in k for $k \in B^+(k_2)$. Finally, unbounded profits imply that there always exists an upper bound $\bar{\mu}_h^3$ such that

$$\delta^F \pi[\tilde{z}(\bar{\mu}_h^3)] = \delta^H \pi[\tilde{z}(1-\bar{\mu}_h^3)].$$

The first and third condition together then imply that there exists a fixed cost threshold k_3 such that $(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0)$ is the equilibrium allocation, and $\Pi_b(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0) < \Pi_h(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0) = \Pi_f(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0)$ for $k \in B^+(k_3)$. This cutoff is defined by

$$k_3 = \delta^F \rho\pi[\tilde{z}(\bar{\mu}_h^3)] + \delta^B \rho(1-\rho)\pi[\tilde{z}^B(1)].$$

Existence of k_4 At k_3, we have already argued that $\Pi_h(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0) = \Pi_f(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0) > 0$. Since $\Pi_h(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0)$ is monotonically decreasing in k, there exists a $k_4 > k_3$ such that profits of the h and f strategy are nil,

$$k_4 = \delta^F \rho\pi[\tilde{z}(\bar{\mu}_h^3)] + \delta^B \rho\pi[\tilde{z}^B(1)].$$

Beyond k_4 For $k \in B^+(k_4)$, it clearly cannot be that $(\bar{\mu}_h^3, 1-\bar{\mu}_h^3, 0)$ is an equilibrium. We first show that, locally, it must be that $\mu_h + \mu_f < 1$ and $\mu_b = 0$. For this to be an equilibrium, it clearly cannot be that $\Pi_h > 0$, for otherwise other firms would enter till either $\Pi_h = 0$ or $\mu_h + \mu_f = 1$. A symmetric argument exists for Π_f. Hence, for $k \in B^+(k_4)$, it must be that $\Pi_h = \Pi_f = 0 > \Pi_b$. Furthermore, when $\mu_h + \mu_f < 1$ and $\mu_b = 0$, the market clearing conditions in state H, F and B respectively read

$$s[z^H(\boldsymbol{\mu})]\rho\mu_h = 1, \quad s[z^F(\boldsymbol{\mu})]\rho\mu_f = 1, \quad s[z^B(\boldsymbol{\mu})](\mu_f + \mu_h)\rho = 1.$$

Let $\zeta(\mu)$ denote the (increasing) function that solves $s[\zeta(\mu)]\rho\mu = 1$. The equilibrium conditions $\Pi_h = \Pi_f = 0$ can then be written as

$$\begin{aligned} \delta^H \rho\pi[\zeta(\mu_h)] + \delta^B \rho\pi[\zeta(\mu_f + \mu_h)] &= k, \\ \delta^F \pi[\zeta(\mu_f)] - \delta^H \pi[\zeta(\mu_h)] &= 0, \\ \mu_f + \mu_h &< 1. \end{aligned}$$

From the second condition, μ_f is increasing in μ_h. From the first condition, the right hand side is increasing in k and the left-hand is decreasing in μ_h. Hence, μ_h is decreasing in k for $k \in B^+(k_4)$. Finally, when profits are unbounded, as k becomes infinitely large, the left-hand side of the first condition has to be large as well, which requires $\mu_h \searrow 0$. From the second condition, $\mu_h \searrow 0$ implies $\mu_f \searrow 0$. Hence, the two conditions above hold jointly for any $k > k_4$, which is depicted in Figure 1.

The proof of Figure 1 holds when profits are unbounded, $\lim_{z \searrow 0} \pi(z) = \infty$. Yet, with HSA preferences and $\varepsilon > 1$, profits may be bounded even as prices tend to zero. When that is the case, there may exist two further thresholds k_5 and k_6 such that, for k between k_5 and k_6, we have $\mu_h > 0$ and $\mu_f = 0$, and for $k > k_6$, no firms enter, $\mu_f = \mu_h = \mu_b = 0$. Furthermore, when profits are bounded, the interval (k_2, k_3) may be empty. However, $k_2 < k_3$ is guaranteed if the difference between γ_F and γ_H is not too large, or alternatively, if $\varepsilon - 1$ is small – in which case profits are necessarily unbounded. Numerically, in Section 5, we do find that $k_2 < k_3$ for relatively large risk premium and elasticity of substitution, namely $\gamma_F/\gamma_H = 0.7$ and $\varepsilon = 1.7$.

Section 3

We begin by deriving the social welfare function in the presence of consumption subsidies that equate consumer prices to marginal costs according to where inputs are sourced.

First, consider the pricing problem facing a producer that pays q per unit for its inputs that faces an aggregator A and that recognizes that consumers will pay only a fraction ν of the sticker price in view of the consumption subsidy at rate $1-\nu$. Then the consumer price of the final product is νp, where p is the producer price. As noted, the government sets the subsidy so that $\nu p = q$, and firms take the subsidy rate as given. They choose the sticker price as

$$p = \arg\max_{\breve{p}} P^{1-\varepsilon} s\left(\frac{\nu \breve{p}}{A}\right)(\nu \breve{p})^{-1}(\breve{p} - q).$$

The solution to this problem yields

$$p = \frac{\sigma(p/A)}{\sigma(p/A) - 1} q.$$

Therefore, the optimal subsidy rates are

$$v\left(z^J\right) = \frac{\sigma\left(z^J\right) - 1}{\sigma\left(z^J\right)},\ J \in \{H, F\},$$

$$v\left(z^{B,i}\right) = \frac{\sigma\left(z^{B,i}\right) - 1}{\sigma\left(z^{B,i}\right)},\ i \in \{H, F\}.$$

These optimal subsidies vary across states of the world if the elasticity of substitution is not constant, and they vary in state B according to the source of the inputs embodied in the final good.

We consider outcomes with $\boldsymbol{\mu} \gg 0$. Now, the market clearing conditions (12) to (14) must still be satisfied, but (27) is replaced with

$$\frac{z^{B,H}}{z^{B,F}} = \lambda := \frac{q_H}{q_F}. \tag{33}$$

It follows that the functions $z^J(\boldsymbol{\mu})$, $J \in \{H, F\}$ are the same as before, but the functions $z^{B,H}(\boldsymbol{\mu})$ and $z^{B,F}(\boldsymbol{\mu})$ are replaced by $\tilde{z}^{B,F}(\boldsymbol{\mu})$ and $\tilde{z}^{B,H}(\boldsymbol{\mu}) \equiv \lambda \tilde{z}^{B,F}(\boldsymbol{\mu})$, where the latter functions are obtained as solutions to (33) and (14). In what follows, we denote with a tilde any function that arise when the consumption subsidies are in place, except for those functions—like $z^H(\boldsymbol{\mu})$ and $z^F(\boldsymbol{\mu})$—that do not change as a result of the subsidies.

With the consumption subsidies in place, firms' operating profits in the various states are

$$\tilde{\pi}^J(\boldsymbol{\mu}) := \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right] - 1} \tilde{P}^J \left[z^J(\boldsymbol{\mu})\right]^{1-\varepsilon},\ J \in \{H, F\}, \tag{34}$$

$$\tilde{\pi}^{B,i}(\boldsymbol{\mu}) := \frac{s\left[\tilde{z}^{B,i}(\boldsymbol{\mu})\right]}{\sigma\left[\tilde{z}^{B,i}(\boldsymbol{\mu})\right] - 1} \tilde{P}^B(\boldsymbol{\mu})^{1-\varepsilon},\ i \in \{H, F\}, \tag{35}$$

where, using (5), the price indexes are

$$\log \tilde{P}^J(\boldsymbol{\mu}) := \log \tilde{P}^J \left[z^J(\boldsymbol{\mu})\right] = C_P + \log \frac{q_J}{z^J(\boldsymbol{\mu})} - n^J(\boldsymbol{\mu}) \int_{z^J(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta, \quad J \in \{H, F\}, \tag{36}$$

$$\log \tilde{P}^B(\boldsymbol{\mu}) = C_P + \log \tilde{A}^B(\boldsymbol{\mu}) - \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}) \int_{z^{B,i}(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta, \tag{37}$$

and $\tilde{A}^B(\boldsymbol{\mu})$ is obtained from

$$1 \equiv n^{B,H}(\boldsymbol{\mu})\, s\left[\frac{q_H}{\tilde{A}^B(\boldsymbol{\mu})}\right] + n^{B,F}(\boldsymbol{\mu})\, s\left[\frac{q_F}{\tilde{A}^B(\boldsymbol{\mu})}\right].$$

Therefore,

$$\tilde{A}^B(\boldsymbol{\mu}) \equiv \frac{q_F}{\tilde{z}^{B,F}(\boldsymbol{\mu})} \equiv \frac{q_F}{\tilde{z}^{B,F}(\boldsymbol{\mu})}. \tag{38}$$

Lump-sum taxes are levied in state J to finance the consumption subsidies paid in that state. Using the subsidy rates $v(z) = [\sigma(z) - 1]/\sigma(z)$, the required taxes are

$$\begin{aligned}
\tilde{T}^H(\boldsymbol{\mu}) &= -(\mu_h + \mu_b)\, \tilde{\pi}^H(\boldsymbol{\mu})\rho, \\
\tilde{T}^F(\boldsymbol{\mu}) &= -(\mu_f + \mu_b)\, \tilde{\pi}^F(\boldsymbol{\mu})\rho, \\
\tilde{T}^B(\boldsymbol{\mu}) &= -(\mu_f + \mu_b)\, \tilde{\pi}^{B,F}(\boldsymbol{\mu})\rho - [\mu_h + \mu_b(1-\rho)]\, \tilde{\pi}^{B,H}(\boldsymbol{\mu})\rho.
\end{aligned}$$

It follows that

$$\sum_{J=H,F,B} \delta^J \tilde{T}^J(\boldsymbol{\mu}) + \sum_{j=h,f,b} \mu_j \tilde{\Pi}_j(\boldsymbol{\mu}) = -(\mu_h + \mu_f + 2\mu_b)\, k.$$

The welfare function (11) therefore becomes

$$\tilde{W}(\boldsymbol{\mu}) = \bar{Y} + \frac{1}{\varepsilon - 1} \sum_{J=H,F,B} \delta^J \tilde{P}^J(\boldsymbol{\mu})^{1-\varepsilon} - (\mu_h + \mu_f + 2\mu_b)\, k. \tag{39}$$

We next characterize the wedges that determine optimal supply chain policies. To this end, we first derive the first-order conditions for the optimal allocation $\boldsymbol{\mu}^o \gg 0$, which are characterized by $\frac{d\tilde{W}(\boldsymbol{\mu}^o)}{d\mu_j} = 0$, $j = h, f$, where, for a general function $G(\boldsymbol{\mu})$, $dG(\boldsymbol{\mu})/d\mu_j$ is the change in $G(\cdot)$ from the variation $d\mu_j = -d\mu_b > 0$. Using the price indexes (36) and (37), together with (12), (13) and

(38), we obtain

$$\frac{d\tilde{W}(\boldsymbol{\mu}^o)}{d\mu_j} = -\sum_{J=H,F} \delta^J \tilde{P}^J(\boldsymbol{\mu}^o)^{1-\varepsilon} \left[\int_{z^J(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \frac{dn^J(\boldsymbol{\mu}^o)}{d\mu_j}$$
$$- \delta^B \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \left[\frac{d\log \tilde{A}^B(\boldsymbol{\mu}^o)}{d\mu_j} + \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}^o) \frac{s\left[\tilde{z}^{B,i}(\boldsymbol{\mu}^o)\right]}{\tilde{z}^{B,i}(\boldsymbol{\mu}^o)} \frac{d\tilde{z}^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j}\right]$$
$$+ \delta^B \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \sum_{i=H,F} \left[\int_{z^{B,i}(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \frac{dn^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j} + k = 0,$$

for $j \in \{h, f\}$. Note, however, that $d\log \tilde{z}^{B,F}(\boldsymbol{\mu}^o)/d\mu_j = d\log \tilde{z}^{B,H}(\boldsymbol{\mu}^o)/d\mu_j$. Then, using (14),

$$\frac{d\log \tilde{A}^B(\boldsymbol{\mu}^o)}{d\mu_j} + \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}) \frac{s\left[\tilde{z}^{B,i}(\boldsymbol{\mu})\right]}{\tilde{z}^{B,i}(\boldsymbol{\mu})} \frac{d\tilde{z}^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j}$$
$$= \quad -\frac{d\log \tilde{z}^{B,F}(\boldsymbol{\mu})}{d\mu_j} \left[1 - \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}) s\left[\tilde{z}^{B,i}(\boldsymbol{\mu})\right]\right] = 0.$$

In other words,

$$\frac{d\log \tilde{P}^B(\boldsymbol{\mu})}{d\mu_j} = -\sum_{i=H,F} \left[\int_{\tilde{z}^{B,i}(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \frac{dn^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j}. \tag{40}$$

The first-order conditions for the first-best allocation can therefore be written as

$$\frac{d\tilde{W}(\boldsymbol{\mu}^o)}{d\mu_j} = -\sum_{J=H,F} \delta^J \tilde{P}^J(\boldsymbol{\mu}^o)^{1-\varepsilon} \left[\int_{z^J(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \frac{dn^J(\boldsymbol{\mu}^o)}{d\mu_j}$$
$$+ \delta^B \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \sum_{i=H,F} \left[\int_{\tilde{z}^{B,i}(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \frac{dn^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j} + k = 0,$$

for $j = h, f$.

Next use $n^F(\boldsymbol{\mu}) = (1-\mu_h)\rho$, $n^H(\boldsymbol{\mu}) = (1-\mu_f)\rho$, $n^{B,F}(\boldsymbol{\mu}) = (\mu_f + \mu_b)\rho$, and $n^{B,H}(\boldsymbol{\mu}) = [\mu_h + \mu_b(1-\rho)]\rho$ to obtain $dn^F(\boldsymbol{\mu})/d\mu_f = 0$, $dn^F(\boldsymbol{\mu})/d\mu_h = -\rho$, $dn^H(\boldsymbol{\mu})/d\mu_h = 0$, $dn^H(\boldsymbol{\mu})/d\mu_f = -\rho$, $dn^{B,F}(\boldsymbol{\mu})/d\mu_f = 0$, $dn^{B,F}(\boldsymbol{\mu})/d\mu_h = -\rho$, $dn^{B,H}(\boldsymbol{\mu})/d\mu_f = -(1-\rho)\rho$, $dn^{B,H}(\boldsymbol{\mu})/d\mu_h = \rho^2$ for $\boldsymbol{\mu} \gg 0$. These expressions allow us to represent $d\tilde{W}(\boldsymbol{\mu}^o)/d\mu_j = 0$ for $j \in \{h, f\}$ as

$$k = \delta^H \tilde{P}^H(\boldsymbol{\mu}^o)^{1-\varepsilon} \left[\int_{z^H(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \rho + \delta^B \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \left[\int_{\tilde{z}^{B,H}(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] (1-\rho)\rho \tag{41}$$

and

$$k=\delta^{F}\tilde{P}^{F}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\left[\int_{z^{F}\left(\boldsymbol{\mu}^{o}\right)}^{\bar{z}}\frac{s\left(\zeta\right)}{\zeta}d\zeta\right]\rho$$
$$+\delta^{B}\tilde{P}^{B}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\left[\int_{\tilde{z}^{B,F}\left(\boldsymbol{\mu}^{o}\right)}^{\bar{z}}\frac{s\left(\zeta\right)}{\zeta}d\zeta-\rho\int_{\tilde{z}^{B,H}\left(\boldsymbol{\mu}^{o}\right)}^{\bar{z}}\frac{s\left(\zeta\right)}{\zeta}d\zeta\right]\rho. \quad (42)$$

By definition,

$$w_{j}^{o}:=\tilde{\Pi}_{j}\left(\boldsymbol{\mu}^{o}\right)-\tilde{\Pi}_{b}\left(\boldsymbol{\mu}^{o}\right)-\frac{d\tilde{W}\left(\boldsymbol{\mu}^{o}\right)}{d\mu_{j}},\quad j\in\{h,f\}.$$

We therefore obtain

$$w_{f}^{o}=k-\delta^{H}\tilde{\pi}^{H}\left(\boldsymbol{\mu}^{o}\right)\rho-\delta^{B}\tilde{\pi}^{B,H}\left(\boldsymbol{\mu}^{o}\right)\left(1-\rho\right)\rho, \quad (43)$$

and

$$w_{h}^{o}=k-\delta^{F}\tilde{\pi}^{F}\left(\boldsymbol{\mu}^{o}\right)\rho-\delta^{B}\left[\tilde{\pi}^{B,F}\left(\boldsymbol{\mu}^{o}\right)-\rho\tilde{\pi}^{B,H}\left(\boldsymbol{\mu}^{o}\right)\right]\rho. \quad (44)$$

Next we use (34), (35) and (41) to derive

$$w_{f}^{o}=\delta^{H}\tilde{P}^{H}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\Phi\left[\tilde{z}^{H}\left(\boldsymbol{\mu}^{o}\right)\right]\rho+\delta^{B}\tilde{P}^{B}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\Phi\left[\tilde{z}^{B,H}\left(\boldsymbol{\mu}^{o}\right)\right]\left(1-\rho\right)\rho, \quad (45)$$

where

$$\Phi\left(z\right):=\int_{z}^{\bar{z}}\frac{s\left(\zeta\right)}{\zeta}d\zeta-\frac{s\left(z\right)}{\sigma\left(z\right)-1},$$

which is equation (17) in the main text. Moreover, using (34), (35) and (42), we obtain

$$\begin{aligned} w_{h}^{o} \;=\; & \delta^{F}\tilde{P}^{F}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\Phi\left[\tilde{z}^{F}\left(\boldsymbol{\mu}^{o}\right)\right]\rho+\delta^{B}\tilde{P}^{B}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\Phi\left[\tilde{z}^{B,H}\left(\boldsymbol{\mu}^{o}\right)\right]\rho\left(1-\rho\right) \\ & +\delta^{B}\tilde{P}^{B}\left(\boldsymbol{\mu}^{o}\right)^{1-\varepsilon}\left\{\Phi\left[\tilde{z}^{B,F}\left(\boldsymbol{\mu}^{o}\right)\right]-\Phi\left[\tilde{z}^{B,H}\left(\boldsymbol{\mu}^{o}\right)\right]\right\}\rho, \end{aligned} \quad (46)$$

which is equation (18) in the main text.

We now want to characterize the absolute and relative sign of these wedges. First, note that (25) implies $\Phi\left(z\right)<0$ under Marshall's Second Law of Demand. Therefore, $w_{f}^{o}<0$. Second,

$$\Phi'\left(z\right)=-\frac{s\left(z\right)}{z}-\frac{s'\left(z\right)}{\sigma\left(z\right)-1}+\frac{s\left(z\right)}{\left[\sigma\left(z\right)-1\right]^{2}}\sigma'\left(z\right)=\frac{s\left(z\right)}{\left[\sigma\left(z\right)-1\right]^{2}}\sigma'\left(z\right)>0.$$

Since $\tilde{z}^{B,H}\left(\boldsymbol{\mu}^{o}\right)=\lambda\tilde{z}^{B,F}\left(\boldsymbol{\mu}^{o}\right)>\tilde{z}^{B,F}\left(\boldsymbol{\mu}^{o}\right)$, this implies $\Phi\left[\tilde{z}^{B,F}\left(\boldsymbol{\mu}^{o}\right)\right]-\Phi\left[\lambda\tilde{z}^{B,F}\left(\boldsymbol{\mu}^{o}\right)\right]<0$ and, therefore, $w_{h}^{o}<0$. These findings are summarized in

Lemma 2 *Let $\sigma'\left(z\right)>0$ for $z\in\left(0,\bar{z}\right)$. Then $w_{j}^{o}<0$ for $j\in\{h,f\}$.*

Now consider two special cases. In the limiting case of symmetric CES preferences, σ is constant and $s(z) := \alpha z^{1-\sigma}$, where $\alpha > 0$ is a constant. In this case $\Phi(z) = 0$ for all z and thus $w_h^o = w_f^o = 0$. That is, the optimal allocation is achieved with no government intervention in the formation of supply chains; i.e., $\varphi_j = 0$ for $j \in \{h, f, b\}$.

In the case of symmetric translog preferences, $s(z) := -\theta \log z$, where $\theta > 0$ is a constant and $z \in (0, 1)$. These preferences imply

$$\int_z^1 \frac{s(\zeta)}{\zeta} d\zeta = \frac{1}{2} \frac{s(z)}{\sigma(z) - 1}.$$

The first-order conditions (41) and (42) become

$$2k = \delta^H \tilde{\pi}^H (\boldsymbol{\mu}^o) \rho + \delta^B \tilde{\pi}^{B,H} (\boldsymbol{\mu}^o) (1 - \rho) \rho,$$

$$2k = \delta^F \tilde{\pi}^F (\boldsymbol{\mu}^o) \rho + \delta^B \left[\tilde{\pi}^{B,F} (\boldsymbol{\mu}^o) - \rho \tilde{\pi}^{B,H} (\boldsymbol{\mu}^o)\right] \rho.$$

Combining these with (43) and (44) yields

$$w_f^o = w_h^o = -k. \tag{47}$$

That is, in the translog case, the optimal allocation is achieved by a policy that subsidizes fully the cost of all investments in single-country supply chains, i.e., $\varphi_b = 0$ and $\varphi_h = \varphi_f = k$.[33] We summarize these findings in

Lemma 3 *(a) In the case of symmetric CES preferences, $w_j^o = 0$ for $j \in \{h, f\}$, which implies that $\varphi_j = 0$ for $j \in \{h, f, b\}$ induces the optimal allocation. (b) In the case of symmetric translog preferences $w_f^o = w_h^o = -k$, which implies that $\varphi_b = 0$ and $\varphi_h = \varphi_f = k$ induces the optimal allocation.*

Finally, consider the difference in the absolute sizes of the wedges. Using (45) and (46), we have

$$\begin{aligned} |w_h^o| - \left|w_f^o\right| = & \ \delta^H \tilde{P}^H (\boldsymbol{\mu}^o)^{1-\varepsilon} \Phi\left[\tilde{z}^H (\boldsymbol{\mu}^o)\right] \rho - \delta^F \tilde{P}^F (\boldsymbol{\mu}^o)^{1-\varepsilon} \Phi\left[\tilde{z}^F (\boldsymbol{\mu}^o)\right] \rho \\ & + \delta^B \tilde{P}^B (\boldsymbol{\mu}^o)^{1-\varepsilon} \left\{\Phi\left[\tilde{z}^{B,H} (\boldsymbol{\mu}^o)\right] - \Phi\left[\tilde{z}^{B,F} (\boldsymbol{\mu}^o)\right]\right\} \rho. \end{aligned} \tag{48}$$

In the limit case $q_H \searrow q_F =: q$, the last term on the right-hand side of this equation equals zero. Moreover, the first-order conditions (41) and (42) imply

$$\delta^H \tilde{P}^H (\boldsymbol{\mu}^o)^{1-\varepsilon} \int_{z^H(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta = \delta^F \tilde{P}^F (\boldsymbol{\mu}^o)^{1-\varepsilon} \int_{z^F(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta. \tag{49}$$

[33] Alternatively, the planner can tax diversification with $\varphi_b = -k$, while leaving $\varphi_h = \varphi_f = 0$.

Therefore,

$$\left|w_h^o\right| - \left|w_f^o\right| = \delta^H \tilde{P}^H \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \frac{s\left[z^H\left(\boldsymbol{\mu}^o\right)\right]}{\sigma\left[z^H\left(\boldsymbol{\mu}^o\right)\right]-1}\rho - \delta^F \tilde{P}^F \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \frac{s\left[z^F\left(\boldsymbol{\mu}^o\right)\right]}{\sigma\left[z^F\left(\boldsymbol{\mu}^o\right)\right]-1}\rho.$$

Using (49), this difference can be expressed as

$$\left|w_h^o\right| - \left|w_f^o\right| = \rho\delta^H \tilde{P}^H \left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \frac{\left\{\frac{s\left[z^H\left(\boldsymbol{\mu}^o\right)\right]}{\sigma\left[z^H\left(\boldsymbol{\mu}^o\right)\right]-1}\right\}\left\{\frac{s\left[z^F\left(\boldsymbol{\mu}^o\right)\right]}{\sigma\left[z^F\left(\boldsymbol{\mu}^o\right)\right]-1}\right\}}{\int_{z^F\left(\boldsymbol{\mu}^o\right)}^{\bar{z}} \frac{s(\zeta)}{\zeta}d\zeta} \left\{\Psi\left[z^F\left(\boldsymbol{\mu}^o\right)\right] - \Psi\left[z^H\left(\boldsymbol{\mu}^o\right)\right]\right\}, \quad (50)$$

where

$$\Psi\left(z\right) := \int_z^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta \Big/ \frac{s\left(z\right)}{\sigma\left(z\right)-1}.$$

We have established

Lemma 4 *Let $q_H \searrow q_F$. Then $\left|w_h^o\right| > \left|w_f^o\right|$ if and only if $\Psi\left[z^F\left(\boldsymbol{\mu}^o\right)\right] > \Psi\left[z^H\left(\boldsymbol{\mu}^o\right)\right]$.*

Next note from (36) that with equal costs in both countries, and $n^J\left(\boldsymbol{\mu}\right) s\left[z^J\left(\boldsymbol{\mu}\right)\right] = 1$,

$$\log \tilde{P}^J\left(\boldsymbol{\mu}\right) = \log \check{P}\left[z^J\left(\boldsymbol{\mu}\right)\right],$$

where

$$\log \check{P}\left(z\right) := C_P + \log\frac{q}{z} - \frac{1}{s\left(z\right)} \int_z^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta.$$

It follows that $\check{P}\left(z\right) \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta$ is a declining function of z. To see this, consider

$$\frac{d}{dz} \log\left\{\check{P}\left(z\right)^{1-\varepsilon}\left[\int_z^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta\right]\right\} = -\left(\varepsilon-1\right)\frac{s'\left(z\right)}{s\left(z\right)^2}\int_z^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta - \frac{s\left(z\right)}{z\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta}.$$

We use

$$\frac{s'\left(z\right)}{s\left(z\right)} = -\frac{\sigma\left(z\right)-1}{z}$$

to obtain

$$\frac{d}{dz} \log\left\{\check{P}\left(z\right)^{1-\varepsilon}\left[\int_z^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta\right]\right\} = \left(\varepsilon-1\right)\frac{\sigma\left(z\right)-1}{zs\left(z\right)}\int_z^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta - \frac{s\left(z\right)}{z\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta}.$$

Finally, from (25), we have

$$\int_z^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta < \frac{s\left(z\right)}{\sigma\left(z\right)-1},$$

which implies

$$\frac{d}{dz}\log\left\{P(z)^{1-\varepsilon}\left[\int_z^{\bar z}\frac{s(\zeta)}{\zeta}d\zeta\right]\right\} = (\varepsilon-1)\frac{\sigma(z)-1}{zs(z)}\int_z^{\bar z}\frac{s(\zeta)}{\zeta}d\zeta - \frac{s(z)}{z\int_z^{\bar z}\frac{s(\zeta)}{\zeta}d\zeta}$$
$$< -\frac{\sigma(z)-\varepsilon}{z} < 0.$$

Applied to (49), this result implies

$$z^H(\boldsymbol{\mu}^o) > z^F(\boldsymbol{\mu}^o). \tag{51}$$

Therefore $|w_h^o| > |w_f^o|$ when $\Psi(z)$ is a decreasing function and $|w_h^o| < |w_f^o|$ when $\Psi(z)$ is an increasing function. We summarize this finding in

Lemma 5 *Let $q_H \searrow q_F$ and $\sigma'(z) > 0$ for $z \in (0,\bar z)$. If $\Psi'(z) < 0$ for all $z \in (0,\bar z)$, then $|w_h^o| > \left|w_f^o\right|$ and if $\Psi'(z) > 0$ for all $z \in (0,\bar z)$, then $|w_h^o| < \left|w_f^o\right|$.*

Section 4

We divide this section in two parts. First, we prove the theoretical foundation of Figure 2. Then, we derive the second-best supply chain policy.

Let $q_F \approx q_H$ and $\gamma_F \approx \gamma_H$ so that both the home and the foreign country are symmetric. When $\boldsymbol{\mu} \gg 0$, all strategies must yield equal expected profits, such that the following conditions must hold jointly

$$\begin{aligned}
\Pi_h(\mu_h,\mu_f,1-\mu_h-\mu_f) &= \Pi_f(\mu_h,\mu_f,1-\mu_h-\mu_f),\\
\Pi_h(\mu_h,\mu_f,1-\mu_h-\mu_f) &= \Pi_b(\mu_h,\mu_f,1-\mu_h-\mu_f),\\
\Pi_f(\mu_h,\mu_f,1-\mu_h-\mu_f) &= \Pi_b(\mu_h,\mu_f,1-\mu_h-\mu_f).
\end{aligned}$$

Using the expressions for expected profits (30) and (31), the first condition rewrites $\pi[z^H(\mu_h,\mu_f,1-\mu_h-\mu_f))] = \pi[z^F(\mu_h,\mu_f,1-\mu_h-\mu_f)]$, where the functiosn z^H and z^F solve, respectively, $s[z^H(\boldsymbol{\mu})]\rho(1-\mu_f) = 1$ and $s[z^F(\boldsymbol{\mu})]\rho(1-\mu_h) = 1$. That is, the functions z^H and z^F are identical. Together with the monotonicity of $z \to \pi(z)$, this implies that $\Pi_h(\boldsymbol{\mu}) = \Pi_f(\boldsymbol{\mu})$ if and only if $\mu_h = \mu_f$.

Using (30) and (32), the second condition is

$$\delta\pi[z^F(\mu_h,\mu_f,1-\mu_h-\mu_f)]\rho + \delta^B\pi[z^B(\mu_h,\mu_f,1-\mu_h-\mu_f)](1-\rho)\rho = k.$$

The function z^F and z^B are given respectively by $s[z^F(\boldsymbol{\mu})]\rho(1-\mu_h) = 1$ and $s[z^B(\boldsymbol{\mu})]\rho[1+(1-\rho)(1-\mu_h-\mu_f)] = 1$, so that z^F is solely a function of μ_h and z^B is solely a function of $\mu_h+\mu_f$.

Totally differentiating the equality above thus yields

$$\left.\frac{d\mu_h}{d\mu_f}\right|_{\Pi_h=\Pi_b} = -\frac{\delta^B d_{\mu_f}\pi[z^B(\mu_h,\mu_f,1-\mu_h-\mu_f)](1-\rho)\rho}{\delta d_{\mu_h}\pi[z^F(\mu_h,\mu_f,1-\mu_h-\mu_f)]\rho+\delta^B d_{\mu_h}\pi[z^B(\mu_h,\mu_f,1-\mu_h-\mu_f)](1-\rho)\rho}.$$

In the above expression, the notation $d_x f[g(x)]$ refers to the total derivative of f with respect to x, $d_x f[g(x)] = f'[g(x)]g'(x)$. Since z^B only depends on $\mu_h+\mu_f$, we have that $d\mu_f\pi(z^B) = d\mu_h\pi(z^B)$. Furthermore, $d\mu_f\pi(z^B) = d\mu_h\pi(z^B) > 0$ and $d\mu_h\pi(z^F) > 0$. Hence, it follows that the curve $\Pi_h = \Pi_b$ slopes downward with a slope in $(-1,0)$. Finally, proceeding similarly with the third condition returns

$$\left.\frac{d\mu_h}{d\mu_f}\right|_{\Pi_f=\Pi_b} = -\frac{\delta d_{\mu_f}\pi[z^H(\mu_h,\mu_f,1-\mu_h-\mu_f)]\rho+\delta^B d_{\mu_f}\pi[z^B(\mu_h,\mu_f,1-\mu_h-\mu_f)](1-\rho)\rho}{\delta^B d_{\mu_h}\pi[z^B(\mu_h,\mu_f,1-\mu_h-\mu_f)](1-\rho)\rho} < -1.$$

These results explain the properties of Figure 2.

We now turn to deriving the general expressions for the wedges w_f and w_h in the constrained optimum, when consumption subsidies are not feasible. We use (11) to calculate $dW(\boldsymbol{\mu})/d\mu_j$. Evaluated at the constrained optimum $\boldsymbol{\mu}^*$, where $dW(\boldsymbol{\mu}^*)/d\mu_j = 0$, we obtain

$$\frac{dW(\boldsymbol{\mu}^*)}{d\mu_j} = \Pi_j(\boldsymbol{\mu}^*) - \Pi_b(\boldsymbol{\mu}^*) + \sum_{i=h,f,b}\mu_i\frac{d\Pi_i(\boldsymbol{\mu}^*)}{d\mu_j} - \sum_{J=H,F,B}\delta^J P^J(\boldsymbol{\mu}^*)^{1-\varepsilon}\frac{d\log P^J(\boldsymbol{\mu}^*)}{d\mu_j} = 0,\ \ j\in\{h,f\}.$$

Rearranging terms, and using the definition of the wedges in the constrained optimum, i.e., $w_j^* = \Pi_j(\boldsymbol{\mu}^*) - \Pi_b(\boldsymbol{\mu}^*)$, yields (19) in the main text. Next, from the expressions for expected profits, (30)-(32), we have

$$\begin{aligned}
\frac{d\Pi_h(\boldsymbol{\mu}^*)}{d\mu_h} &= \delta^B\rho\frac{d\pi^{B,H}(\boldsymbol{\mu}^*)}{d\mu_h},\\
\frac{d\Pi_h(\boldsymbol{\mu}^*)}{d\mu_f} &= \delta^H\rho\frac{\partial\pi\left[z^H(\boldsymbol{\mu}^*)\right]}{\partial z}\frac{\partial z^H(\boldsymbol{\mu}^*)}{\partial\mu_f} + \delta^B\rho\frac{d\pi^{B,H}(\boldsymbol{\mu}^*)}{d\mu_f},\\
\frac{d\Pi_f(\boldsymbol{\mu}^*)}{d\mu_f} &= \delta^B\rho\frac{d\pi^{B,F}(\boldsymbol{\mu}^*)}{d\mu_f},\\
\frac{d\Pi_f(\boldsymbol{\mu}^*)}{d\mu_h} &= \delta^F\rho\frac{\partial\pi\left[z^F(\boldsymbol{\mu}^*)\right]}{\partial z}\frac{dz^F(\boldsymbol{\mu}^*)}{d\mu_f} + \delta^B\rho\frac{d\pi^{B,F}(\boldsymbol{\mu}^*)}{d\mu_h},\\
\frac{d\Pi_b(\boldsymbol{\mu}^*)}{d\mu_h} &= \delta^F\rho\frac{\partial\pi\left[z^F(\boldsymbol{\mu}^*),q_F\right]}{\partial z}\frac{dz^F(\boldsymbol{\mu}^*)}{d\mu_h} + \delta^B\left[\rho\frac{d\pi^{B,F}(\boldsymbol{\mu}^*)}{d\mu_h} + \rho(1-\rho)\frac{d\pi^{B,H}(\boldsymbol{\mu}^*)}{d\mu_h}\right],\\
\frac{d\Pi_b(\boldsymbol{\mu}^*)}{d\mu_f} &= \delta^H\rho\frac{\partial\pi\left[z^H(\boldsymbol{\mu}^*),q_H\right]}{\partial z}\frac{dz^H(\boldsymbol{\mu}^*)}{d\mu_f} + \delta^B\left[\rho\frac{d\pi^{B,F}(\boldsymbol{\mu}^*)}{d\mu_f} + \rho(1-\rho)\frac{d\pi^{B,H}(\boldsymbol{\mu}^*)}{d\mu_f}\right].
\end{aligned}$$

Substituting these derivatives into the expression for the wedges (19), we obtain

$$
\begin{aligned}
w_j^* = -\delta^K & \left\{ \frac{1}{\sigma[z^K(\boldsymbol{\mu}^*)]} \frac{\partial \log \pi^K[z^K(\boldsymbol{\mu}^*)]}{\partial z} - \frac{\partial \log P^K[z^K(\boldsymbol{\mu}^*)]}{\partial z} \right\} P^K[z^K(\boldsymbol{\mu}^*)]^{1-\varepsilon} \frac{dz^K(\boldsymbol{\mu}^*)}{d\mu_j} \\
& - \delta^B \left\{ \sum_K \frac{n^{B,K}(\boldsymbol{\mu}^*) s[z^{B,K}(\boldsymbol{\mu}^*)]}{\sigma[z^{B,K}(\boldsymbol{\mu}^*)]} \frac{d \log \pi^{B,K}(\boldsymbol{\mu}^*)}{d\mu_j} - \frac{d \log P^B(\boldsymbol{\mu}^*)}{d\mu_j} \right\} P^B(\boldsymbol{\mu}^*)^{1-\varepsilon}, \quad (52)
\end{aligned}
$$

where $K = F$ if $j = h$ and $K = H$ if $j = f$. The first term on the right-hand side of (52) represents the net externality in state K, i.e., the business-stealing externality combined with the consumer-surplus externality. The second term represents the net externality in state B.

To compute these wedges, we need explicit expressions for the partial derivatives in (52). First note that the expressions for the semi-elasticities of the price index and profits in state $K \in \{H, F\}$ are given by (23) and (24), respectively. For state B, differentiate the expression for relative prices (27) to obtain

$$
\frac{d \log z^{B,H}(\boldsymbol{\mu})}{d\mu_j} \Big/ \frac{d \log z^{B,F}(\boldsymbol{\mu})}{d\mu_j} = \left\{ 1 - z^{B,F}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,F}(\boldsymbol{\mu})]}{\partial z} \right\} \Big/ \left\{ 1 - z^{B,H}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,H}(\boldsymbol{\mu})]}{\partial z} \right\},
$$

where $\eta(z) := \sigma(z)/(\sigma(z) - 1)$ is the markup factor. Together with condition (14), we obtain

$$
\frac{d \log z^{B,K}(\boldsymbol{\mu})}{d\mu_h} = - \frac{\rho s[z^{B,F}(\boldsymbol{\mu})] - \rho^2 s[z^{B,H}(\boldsymbol{\mu})]}{\phi(\boldsymbol{\mu}) \left\{ 1 - z^{B,K}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,K}(\boldsymbol{\mu})]}{\partial z} \right\}}, \; K \in \{H, F\} \quad (53)
$$

and

$$
\frac{d \log z^{B,K}(\boldsymbol{\mu})}{d\mu_f} = - \frac{\rho(1-\rho) s[z^{B,H}(\boldsymbol{\mu})]}{\phi(\boldsymbol{\mu}) \left\{ 1 - z^{B,K}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,K}(\boldsymbol{\mu})]}{\partial z} \right\}}, \; K \in \{H, F\}, \quad (54)
$$

where

$$
\phi(\boldsymbol{\mu}) := \sum_{L=H,F} n^{B,L}(\boldsymbol{\mu}) s\left[z^{B,L}(\boldsymbol{\mu})\right] \left\{ \frac{\sigma[z^{B,L}(\boldsymbol{\mu})] - 1}{1 - z^{B,L}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,L}(\boldsymbol{\mu})]}{\partial z}} \right\}.
$$

Differentiating the price index in state B, (28), we obtain

$$
\begin{aligned}
\frac{d \log P^B(\boldsymbol{\mu})}{d\mu_h} = & \frac{d \log z^{B,H}(\boldsymbol{\mu})}{d\mu_h} \frac{\partial \log \eta[z^{B,H}(\boldsymbol{\mu})]}{\partial \log z} \\
& + n^{B,F}(\boldsymbol{\mu}) s[z^{B,F}(\boldsymbol{\mu})] \left[\frac{d \log z^{B,F}(\boldsymbol{\mu})}{d\mu_h} - \frac{d \log z^{B,H}(\boldsymbol{\mu})}{d\mu_h} \right] \\
& + \rho \left[\int_{z^{B,F}(\boldsymbol{\mu})}^{z^{B,H}(\boldsymbol{\mu})} \frac{s(\zeta)}{\zeta} d\zeta + (1-\rho) \int_{z^{B,H}(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta \right], \quad (55)
\end{aligned}
$$

and

$$\frac{d\log P^B(\boldsymbol{\mu})}{d\mu_f} = \frac{d\log z^{B,F}(\boldsymbol{\mu})}{d\mu_f}\frac{\partial \log \eta[z^{B,F}(\boldsymbol{\mu})]}{\partial \log z}$$
$$+ n^{B,H}(\boldsymbol{\mu})s[z^{B,H}(\boldsymbol{\mu})]\left[\frac{d\log z^{B,H}(\boldsymbol{\mu})}{d\mu_f} - \frac{d\log z^{B,F}(\boldsymbol{\mu})}{d\mu_f}\right]$$
$$+ \rho(1-\rho)\int_{z^{B,H}(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta}d\zeta. \tag{56}$$

Finally, the change in profits is given by

$$\frac{d\log \pi^{B,K}(\boldsymbol{\mu})}{d\mu_j} = -\left(\sigma[z^{B,K}(\boldsymbol{\mu})] - 1 + \frac{\partial \log \sigma[z^{B,K}(\boldsymbol{\mu})]}{\partial \log z}\right)\frac{d\log z^{B,K}(\boldsymbol{\mu})}{d\mu_j}$$
$$-(\varepsilon - 1)\frac{d\log P^B(\boldsymbol{\mu})}{d\mu_j}, \; j\in\{h,f\}, \; K\in\{H,F\}. \tag{57}$$

To better understand these expressions, we consider the symmetric limiting case where $q_H \approx q_F = q$ and $\gamma_H \approx \gamma_F = \gamma$. In this setting, $\delta^F \approx \delta^H = \delta$, and $z^{B,F}(\boldsymbol{\mu}) \approx z^{B,H}(\boldsymbol{\mu}) =: z^B(\boldsymbol{\mu})$. As a result, the expression for the wedge (52) becomes

$$w_j^* = -\delta\left\{\frac{\frac{\partial \log \pi[z^K(\boldsymbol{\mu}^*)]}{\partial z}}{\sigma\left[z\left(\boldsymbol{\mu}^*\right)\right]} - \frac{\partial \log P[z^K(\boldsymbol{\mu}^*)]}{\partial z}\right\}P[z^K(\boldsymbol{\mu}^*)]^{1-\varepsilon}\frac{dz^K(\boldsymbol{\mu}^*)}{d\mu_j}$$
$$-\delta^B\left\{\frac{\frac{\partial \log \pi[z^B(\boldsymbol{\mu}^*),q]}{\partial z}}{\sigma\left[z^B\left(\boldsymbol{\mu}^*\right)\right]} - \frac{\partial \log P[z^B(\boldsymbol{\mu}^*)]}{\partial z}\right\}P[z^B(\boldsymbol{\mu}^*)]^{1-\varepsilon}\frac{dz^B(\boldsymbol{\mu}^*)}{d\mu_j}. \tag{58}$$

The term $\partial \log P/\partial z$ represents the consumer-surplus externality, whereas the term $\left(\partial \log \pi/\partial z\right)/\sigma$ represents the business-stealing externality.

Before considering the signs of the wedges, we need to show that $\mu_h^* = \mu_f^*$. Recall that the necessary first-order conditions for an interior allocation are

$$W_j(\boldsymbol{\mu}^*) = \Pi_j(\boldsymbol{\mu}^*) - \Pi_b(\boldsymbol{\mu}^*) - w_j^* = 0, \quad j = h, f.$$

For these two necessary conditions to hold jointly, it must be that

$$\Pi_h(\boldsymbol{\mu}^*) - w_h^* = \Pi_f(\boldsymbol{\mu}^*) - w_f^*.$$

Using (58) and the expressions for expected profits, we find that this equality indeed holds when $\mu_h^* = \mu_f^*$. This allocation corresponds to the unique optimal constrained allocation if W is globally concave. Proving the global concavity of W for general HSA preferences turns out to be a tricky task. Instead, we now show that $\mu_h^* = \mu_f^*$ is an optimum when preferences are symmetric translog. Additionally, we prove at the end of this Section that W is indeed globally concave when preferences are CES.

To prove that $\mu_h^* = \mu_f^*$ is an optimum when preferences are symmetric translog, we show that increasing μ_f is welfare-improving if and only if $\mu_f < \mu_h$. Specifically, we consider the variation $d\mu = (d\mu_h, d\mu_f, 0)$ with $d\mu_h = -d\mu_f$. Totally differentiating the welfare function (11) and imposing $d\mu_h - d\mu_f$ returns

$$\frac{dW(\boldsymbol{\mu})}{d\mu_f} \propto \frac{\partial\Omega(\mu_h)}{\partial\mu_h} - \frac{\partial\Omega(\mu_f)}{\partial\mu_f},$$

where

$$\Omega(\mu) := -\left(n(\mu)\pi[z(\mu)] + \frac{1}{\varepsilon - 1}P[z(\mu)]^{1-\varepsilon}\right),$$

and $n(\mu) = \rho(1-\mu)$, the function z solves $s[z(\mu)]n(\mu) = 1$, P is given by (21) and $\pi(z)$ by (22). When preferences are symmetric translog, $s(z) = -\theta\log(z)$, and the function Ω becomes[34]

$$\Omega(\mu) \propto -\left(\frac{1}{1+\theta n(\mu)} + \frac{1}{\varepsilon - 1}\right)\left(\frac{1+\theta n(\mu)}{\theta n(\mu)}\right)^{1-\varepsilon} \exp\left(\frac{1-\varepsilon}{2\theta n(\mu)}\right).$$

The function Ω is convex as long as $\sigma(\mu) = 1 + \theta n(\mu) > \varepsilon$, which holds through Assumption 2. Hence, $\partial_\mu\Omega(\mu)$ is increasing, and $d_{\mu_f}W(\boldsymbol{\mu}) > 0 \iff \partial_{\mu_h}\Omega(\mu_h) > \partial_{\mu_f}\Omega(\mu_f) \iff \mu_h > \mu_f$.

With these results in mind, we turn to signing the wedges. Since $\mu_h^* = \mu_f^* =: \mu^*$, the wedges for the two sole-sourcing strategies are equal, i.e., $w_h^* = w_f^* =: w^*$. Furthermore, from (58), we have

$$w^* > 0 \quad \text{if} \quad \frac{\partial\log\pi(z)}{\partial z} > \sigma(z)\frac{\partial\log P(z)}{\partial z} \quad \text{for } z \in \{z^K(\mu^*), z^B(\mu^*)\},$$

and

$$w^\star < 0 \quad \text{if} \quad \frac{\partial\log\pi(z)}{\partial z} < \sigma(z)\frac{\partial\log P(z)}{\partial z} \quad \text{for } z \in \{z^K(\mu^*), z^B(\mu^*)\},$$

which follows from the fact that z^K and z^B are decreasing in μ_j. General HSA preferences do not yield simple parametric conditions that satisfy these inequalities. But we can gain further insight by considering the special cases of CES preferences and translog preferences.

First, with symmetric CES preferences, $s(z) = \alpha z^{1-\sigma}$ and $\sigma(z) = \sigma$ is a constant. Using (23) with this market-share function, the consumer-surplus externality becomes

$$\frac{\partial\log P(z)}{\partial z} = \frac{s'(z)}{s(z)}\frac{1}{\sigma - 1} < 0.$$

Next, using (24), the business-stealing externality simplifies to

$$\frac{\partial\log\pi(z)}{\partial z} = \frac{s'(z)}{s(z)}\frac{\sigma - \varepsilon}{\sigma - 1} < 0.$$

[34] Recall that under symmetric translog preferences, the elasticity of substitution is $\sigma(z) = 1 - 1/\log(z)$, the market clearing condition implies $\log z(\mu) = -1/[\theta\rho(1-\mu)]$, and finally $[1/s(z)]\int_z^1 s(\zeta)/\zeta d\zeta = -\log(z)/2$.

Together they imply

$$\frac{\partial \log \pi(z)}{\partial z} - \sigma \frac{\partial \log P(z)}{\partial z} = -\frac{s'(z)}{s(z)} \frac{\varepsilon}{\sigma - 1} > 0 \text{ for all } z.$$

We have established

Lemma 6 *Let* $q_H \searrow q_F$*,* $\gamma_H \searrow \gamma_F$*, and let consumers hold symmetric CES preferences. Then,* $w_h^* = w_f^* > 0$.

Turning to symmetric translog preferences, let $s(z) = -\theta \log(z)$ for $z \in (0, 1)$. Now (23) implies

$$\frac{\partial \log P(z)}{\partial \log z} = \frac{1}{\log z - 1} - \frac{1}{2}$$

while (24) implies

$$\frac{\partial \log \pi(z)}{\partial \log z} = \left(1 - \frac{1}{\log z} - \varepsilon\right)\left(\frac{1}{\log z - 1} - \frac{1}{2}\right) + \frac{1}{2 \log z}.$$

Together, these two expressions imply

$$\frac{\partial \log \pi(z)}{\partial \log z} - \sigma(z) \frac{\partial \log P(z)}{\partial \log z} = \varepsilon + \frac{1}{\log z} \frac{\log z - 1}{\log z - 3}.$$

Under symmetric translog preferences, the adding up constraints of market shares generate relative prices $\log z^J(\mu) = -1/[\theta n(\mu)]$ for $n^J(\mu) = n(\mu) := \rho(1-\mu)$, $J \in \{H, F\}$, and $\log z^B(\mu) = -1/[\theta n^B(\mu)]$ for $n^B(\mu) := \rho[1 + (1-\rho)(1-2\mu)]$. It follows that

$$\frac{\partial \log \pi[z(\mu^*)]}{\partial z} > \sigma[z(\mu^*)] \frac{\partial \log P[z(\mu^*)]}{\partial z} \quad \Longleftrightarrow \quad \varepsilon > \theta n^K(\mu^*) \frac{1 + \theta n^K(\mu^*)}{1 + 3\theta n^K(\mu^*)}, \quad K \in \{H, F, B\}.$$

Finally, we note that $n^B(\mu) > n(\mu)$ for $\mu < 1/2$, and that the product $x(1+x)/(1+3x)$ is increasing in x. We conclude that

$$\varepsilon < \theta n(\mu^*) \frac{1 + \theta n(\mu^*)}{1 + 3\theta n(\mu^*)} \Longrightarrow w^* < 0,$$

and

$$\varepsilon > \theta n^B(\mu^*) \frac{1 + \theta n^B(\mu^*)}{1 + 3\theta n^B(\mu^*)} \Longrightarrow w^* > 0.$$

Although the values of $n(\mu^*)$ and $n^B(\mu^*)$ are endogenous, it is possible to derive parametric restrictions that guarantee that one or the other of these inequalities holds. Specifically, if $\varepsilon < \theta n(\mu^*) \frac{1+\theta n(\mu^*)}{1+3\theta n(\mu^*)}$ holds for the smallest possible value of n, then it must hold for all n. Therefore $w^* < 0$ if $\varepsilon < \theta\rho(2+\theta\rho)/2(2+3\theta\rho)$. Similarly, if $\varepsilon > \theta n^B(\mu^*) \frac{1+\theta n^B(\mu^*)}{1+3\theta n^B(\mu^*)}$ holds for the largest possible value of n^B, then it must hold for all n. Therefore $w^* > 0$ if $\varepsilon > \theta\rho(2-\rho)[1+\theta\rho(2-\rho))/(1+3\theta\rho(2-\rho)]$.[35]

[35]Technically, we also need to ensure that $\min\{\sigma[z(\mu^*)], \sigma[z^B(\mu^*)]\} = \sigma[z(\mu^*)] = 1 + \theta n(\mu^*) > \varepsilon$. This is not a

Lemma 7 *Let $q_H \searrow q_F$, $\gamma_H \searrow \gamma_F$ and suppose that consumers have symmetric translog preferences. Then*

$$\varepsilon < \frac{\theta\rho(2+\theta\rho)}{2(2+3\theta\rho)} \Longrightarrow w^* < 0,$$

and

$$\varepsilon > \frac{\theta\rho(2-\rho)\left[1+\theta\rho(2-\rho)\right]}{1+3\theta\rho(2-\rho)} \Longrightarrow w^* > 0.$$

To conclude this section, we return to the special case of CES preferences to show that Lemma 6 generalizes to settings with asymmetric costs and risks. Returning to (52), we have already shown that the first term in parenthesis is negative. In state B, constant mark-ups simplify equations (53) and (54) to

$$\frac{d\log z^{B,K}}{d\mu_h} = -\frac{\rho\left\{s[z^{B,F}(\boldsymbol{\mu}^*)] - \rho s[z^{B,H}(\boldsymbol{\mu}^*)]\right\}}{\sigma-1} < 0,\ K \in \{H,F\},$$

and

$$\frac{d\log z^{B,K}}{d\mu_f} = -\frac{\rho(1-\rho)s[z^{B,H}(\boldsymbol{\mu}^*)]}{\sigma-1} < 0,\ K \in \{H,F\}.$$

Furthermore, the semi-elasticity of the price index (55) and (56) becomes

$$\frac{d\log P^B(\boldsymbol{\mu}^*)}{d\mu_j} = -\frac{d\log z^{B,H}(\boldsymbol{\mu}^*)}{d\mu_j} = -\frac{d\log z^{B,F}(\boldsymbol{\mu}^*)}{d\mu_j} > 0,\ j \in \{h,f\}.$$

Similarly, the semi-elasticity of profits (57) becomes

$$\frac{d\log \pi^{B,K}(\boldsymbol{\mu}^*)}{d\mu_j} = -(\sigma-\varepsilon)\frac{d\log z^{B,F}(\boldsymbol{\mu}^*)}{d\mu_j} = -(\sigma-\varepsilon)\frac{d\log z^{B,H}(\boldsymbol{\mu}^*)}{d\mu_j} > 0,\ j \in \{h,f\}.$$

Combining these expressions, the second term in (52) becomes

$$\begin{aligned}
&\sum_{K=H,f} \frac{n^{B,K}(\boldsymbol{\mu}^*)s[z^{B,K}(\boldsymbol{\mu}^*)]}{\sigma}\frac{d\log \pi^{B,K}(\boldsymbol{\mu}^*)}{d\mu_j} - \frac{d\log P^B(\boldsymbol{\mu}^*)}{d\mu_j} \\
= \quad & \frac{\varepsilon}{\sigma}\frac{d\log z^{B,K}(\boldsymbol{\mu}^*)}{d\mu_j} < 0,\ j \in \{h,f\} \text{ and } K \in \{H,F\}.
\end{aligned}$$

Then (52) implies

$$\begin{aligned}
w_h^* &= \rho\left(\frac{\varepsilon}{\sigma-1}\right)\left(\delta^F \pi[z^F(\boldsymbol{\mu}^*)] + \delta^B\left\{\pi[z^{B,F}(\boldsymbol{\mu}^*)] - \rho\pi[z^{B,H}(\boldsymbol{\mu}^*)]\right\}\right), \\
w_f^* &= \rho\left(\frac{\varepsilon}{\sigma-1}\right)\left\{\delta^H \pi[z^H(\boldsymbol{\mu}^*)] + \delta^B(1-\rho)\pi[z^{B,H}(\boldsymbol{\mu}^*)]\right\}.
\end{aligned}$$

Together with the planner's first-order conditions, these expressions yield

concern for the sufficient condition $\varepsilon < \theta n(\mu^*)\frac{1+\theta n(\mu^*)}{1+3\theta n(\mu^*)}$. Regarding $\varepsilon > \theta n^B(\mu^*)\frac{1+\theta n^B(\mu^*)}{1+3\theta n^B(\mu^*)}$, a sufficient condition for $\sigma[z(\mu^*)] > \varepsilon$ for all μ^* is $1+\theta n(1/2) = 1+\theta\rho/2 > \varepsilon$ since n is decreasing in μ.

Lemma 8 *Suppose consumers have symmetric CES preferences. Then*

$$w_h^* = w_f^* = \left(\frac{\varepsilon}{\sigma+\varepsilon-1}\right) k > 0.$$

Evidently, in the CES case, the two wedges are positive and equal to one another, which implies that the constrained optimum can be achieved with a subsidy for diversification, i.e., $\varphi_b > 0$, with $\varphi_h = \varphi_f = 0$.

Finally, we conclude this section by showing that the first order conditions are necessary and sufficient when preferences are CES – that is, that the welfare function W is globally concave. When preferences are symmetric CES, the price index (21) in state $J \in \{H, F\}$ simplifies to

$$P[z^J(\boldsymbol{\mu}), q_J] = \frac{q_J}{z^J(\boldsymbol{\mu})} = n^J(\boldsymbol{\mu})^{\frac{1}{1-\sigma}} \cdot q_J, \tag{59}$$

while the price index in state B becomes

$$P^B(\boldsymbol{\mu}) = \left(\sum_{J=H,F} n^{B,J}(\boldsymbol{\mu}) q_J^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{60}$$

Additionally, the profit of an active firm in state $J \in \{H, F\}$ is

$$\pi[z^J(\boldsymbol{\mu}), q_J] = \left(\frac{q_J^{1-\varepsilon}}{\sigma}\right) n^J(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}}.$$

and the profit of an active firm in state B purchasing an input from country $J \in \{H, F\}$ is

$$\pi^{B,J}(\boldsymbol{\mu}) = \left(\frac{q_J^{1-\sigma}}{\sigma}\right) \left(\sum_{\ell=H,F} n^{B,\ell}(\boldsymbol{\mu}) q_\ell^{1-\sigma}\right)^{\frac{\varepsilon-\sigma}{\sigma-1}}.$$

Under this special functional form, when the allocation is interior, $\mu_f > 0$, $\mu_h > 0$ and $\mu_b = 1 - \mu_f - \mu_h > 0$, the welfare function then simplifies to

$$W(\mu_h, \mu_f) = c \sum_{J=H,F,B} \delta^J P^J(\boldsymbol{\mu})^{1-\varepsilon} - k(2 - \mu_h - \mu_f),$$

where $c := 1/\sigma + 1/(\varepsilon - 1)$. Plugging in the expression for the price indices, (59) and (60), we have

$$W(\mu_h, \mu_f) = c \left[\sum_{J=H,F} \delta^J n^J(\boldsymbol{\mu})^{\frac{1-\varepsilon}{1-\sigma}} q_J^{1-\varepsilon} + \delta^B \left(\sum_{\ell=H,F} n^{B,\ell}(\boldsymbol{\mu}) q_\ell^{1-\sigma}\right)^{\frac{1-\varepsilon}{1-\sigma}}\right] - k(2 - \mu_h - \mu_f).$$

Double differentiating W, we obtain that the elements of the Hessian matrix are

$$\frac{\partial^2 W(\mu_h,\mu_f)}{\partial(\mu_f)^2} \propto -\left(\delta^H n^H(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}-1} q_H^{1-\varepsilon} + \delta^B (1-\rho)^2 q_H^{2(1-\sigma)} P^B(\boldsymbol{\mu})^{2\sigma-1-\varepsilon}\right) < 0,$$
$$\frac{\partial^2 W(\mu_h,\mu_f)}{\partial(\mu_h)^2} \propto -\left(\delta^F n^F(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}-1} q_F^{1-\varepsilon} + \delta^B \left[q_F^{1-\sigma} - \rho q_H^{1-\sigma}\right]^2 P^B(\boldsymbol{\mu})^{2\sigma-1-\varepsilon}\right) < 0,$$
$$\frac{\partial^2 W(\mu_h,\mu_f)}{\partial\mu_f \partial\mu_h} \propto -\delta^B (1-\rho) q_H^{1-\sigma} \left[(q_F^{1-\sigma} - \rho q_H^{1-\sigma}\right] P^B(\boldsymbol{\mu})^{2\sigma-1-\varepsilon} < 0,$$

where the constant of proportionality is the same. Inspection of the Hessian matrix shows that W is globally concave.

Section 5

In this section, we extend the numerical results of Section 6 in the main text to include simulations with both asymmetric risks *and* costs. Figure 6 extends the comparative statics of panels (c) and (d) in Figure 4 by comparing the effect of cross-country differences in risk on the optimal supply-chain policies under two different cost discounts.[36] Panels (a) and (b) in Figure 6 depict, respectively, the fraction of firms that adopt a particular supply chain strategy and the optimal policy under the symmetric cost simulation of Figure 4. Panels (c) and (d) plot the same variables for a positive cost discount of 5%.

When the cost discount is large but the risk premium is minimal, offshoring is *ceteris paribus* more profitable than onshoring, and firms locate their supply chains disproportionately in the foreign country, both in the equilibrium and in the constrained optimum. The wedges remain positive for both strategies, although they are no longer equal. Indeed, as discussed in Section 5, when risks are identical across countries but the input cost is lower in the foreign country, the social planner wants to tax relatively more the exclusive offshore relationships as the price index is lower in state F.

As in the case with symmetric costs depicted in Figure 4, when the cost differential is positive, an increase in the risk premium is associated with a greater fraction of diversified firms, a greater fraction of firms that form relationships only onshore, and a smaller fraction of firms that form relationships only abroad. However, in this case, firms face a tension between safe-but-expensive and riskier-but-cheaper suppliers. In panel (c), we see that for risk differentials greater than 10%, a cost discount of 5% is no longer enough to favor offshore investments, and firms locate disproportionately their supply chains in the safe-but-expensive home country.

Qualitatively, the effect of an increase in the risk premium on the optimal policies when the cost differential is positive also mimics what we have seen for symmetric costs. As the foreign risk increases, relatively more firms locate their supply chains exclusively in the home country, which triggers a relative increase in the price index in state F compared to state H, and with it an increase

[36]The effect of a positive cost differential on the comparative statics for the risk premium is qualitatively similar for the cases of $\varepsilon = 1.2$ and $\varepsilon = 1.7$. To conserve space, we present only the latter.

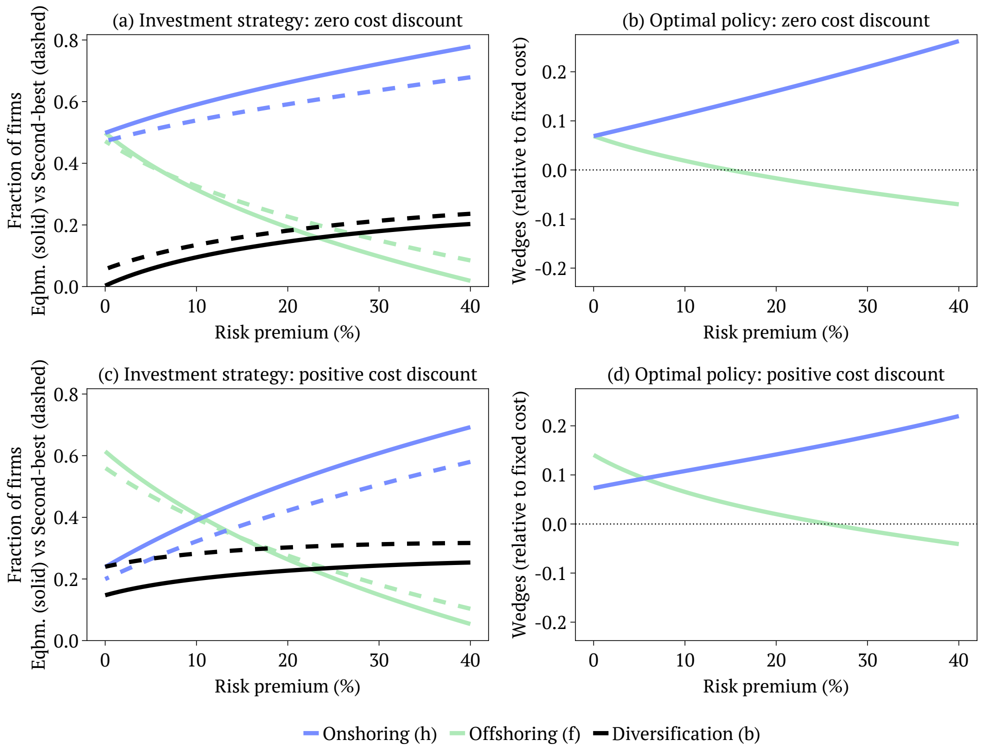

Figure 6: Second-Best Policies: Risk Differences Across Locations with Two Cost Scenarios

Note: Baseline simulation is $\varepsilon = 1.7$, $\gamma_H = \gamma_F = 0.9$, $q_H = q_F = 0.1$, $\theta = 8.0$, and $\rho = 0.7$. Fixed cost chosen so that $\min(\mu_b^\star, \mu_b^o) \approx 0$ in the baseline symmetric simulation. This yields $k = 0.37$. The risk premium is computed as $-(\gamma_F - \gamma_H)/\gamma_H$, where we keep γ_H constant at its baseline value. The cost discount in panels (b) and (d) is 5%.

in w_h^* but a decrease in w_f^*. Compared with the symmetric cost simulation, the difference is now that the price index was initially lower in state F relative to state H due to the lower input cost in the foreign country. Thus, an increase in foreign risk initially shrinks the market's misallocation between home sourcing and foreign sourcing, and the wedges converge for a risk premium of 5%. Then, as the risk premium continues to grow, the price index in state F continues to increase, and the planner wants to tax relatively more the exclusive onshore relationships. Finally, for a sufficiently large risk premium, the planner's desire to shift the location of exclusive-sourcing from the home country to the foreign country implies again a tax on onshore relationships but a subsidy for investing in a single relationship abroad.

Figure 7 extends the comparative statics of panels (c) and (d) in Figure 5 by allowing for a positive risk premium. Panel (a) and (b) reproduce the results illustrated in panels (c) and (d) of the earlier figure, where $\gamma_H = \gamma_F$, while panels (c) and (d) in Figure 7 depict outcomes and policies with a positive risk premium of 15%.[37] When the cost discount is small relative to the risk premium, onshore sourcing relationships are relatively more attractive and a larger fraction of firms opt for strategy h. As the cost discount grows, the relative advantage of the foreign country

[37]Once again the qualitative properties of the figure are similar for $\varepsilon = 1.2$ and $\varepsilon = 1.7$, so we present only the latter.

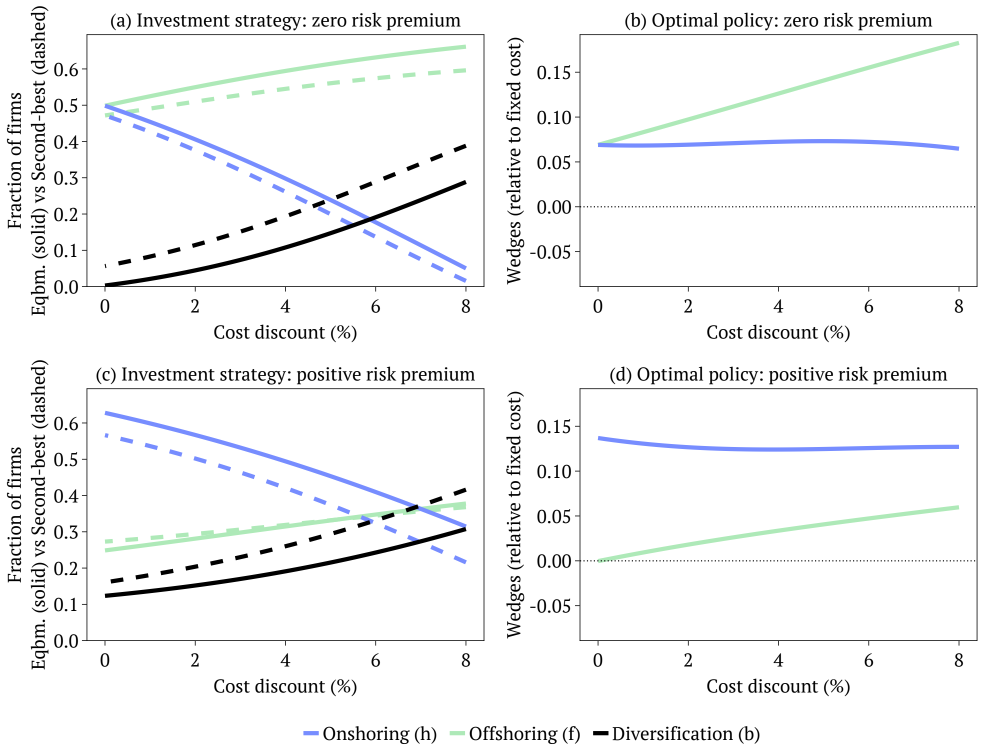

Figure 7: Second-Best Policies: Cost Differences Across Locations with Two Risk Scenarios

Note: Baseline simulation is $\varepsilon = 1.7$, $\gamma_H = \gamma_F = 0.9$, $q_H = q_F = 0.1$, $\theta = 8.0$, and $\rho = 0.7$. Fixed cost chosen so that $\min(\mu_b^\star, \mu_b^o) \approx 0$ in the baseline symmetric simulation. This yields $k = 0.37$. The cost discount is computed as $-(q_F - q_H)/q_H$, where we keep q_H constant at its baseline value. The risk premium in panels (b) and (d) is 15%.

increases, and a larger fraction of firms decide to form their exclusive relationship with foreign suppliers. This intuitive pattern mimics the findings for the case where risks are symmetric.

Regarding the optimal policies, the wedge for strategy f is relatively smaller than that for strategy h when the cost discount is relatively small. This echoes the discussion of Figure 4; when the risk premium is large but the cost discount is small, the monopoly distortion is more severe in state F when the price index is higher, and the planner wishes to combat the higher prices in this state with a policy that tilts sourcing towards the foreign country. As the cost discount grows further, the fraction of firms that form exclusive relationships with foreign supplier rises, which, as in the scenario with symmetric risks, reduces the social benefit from promoting consumption in state F, and thus the gap between w_f^* and w_h^*.

We have explored a large variety of parameters besides those illustrated here. In general, the optimal policies hinge on which country is more attractive for exclusive sourcing based on the tradeoff between risk and cost and the implications of these asymmetric investments on the sizes of the monopoly distortions in the various states of the world.